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The Mathematics of Mixers: Basic Principles

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This month's tutorial is a first introduction to the mathematical principles that describe the operation of frequency mixers

Mixers are classic RF/microwave circuits that make it possible to translate RF signals from one frequency to another. Ideally, they implement this frequency change with no effect on the amplitude and frequency components of the signal's modulation.

Frequency Translation

Mixers are nonlinear circuits; they rely on near-perfect nonlinearity. This sounds like a contradiction, but it means that perfect switching—discontinuity being the ultimate nonlinearity—will result in ideal mixer behavior. We will describe how this switching takes place in a circuit later on, but first let's review the overall behavior of the mixing process.

Nonlinear response creates new signals where none previously existed. In the case of two unmodulated signals applied to the input of a nonlinear device, there will be a series of output signals that contain multiples of the input signals (harmonics), plus sums and differences of ALL signals, fundamental and harmonic, as described by [1]:

$$f_{out} = |nf_1 \pm mf_2|$$

where f_{out} represents all output signals, f_1 and f_2 are the two input signals, n and m are the order of the harmonics, from zero (fundamental) to infinity.

Mathematically, this is an infinite Fourier type of series, where the amplitude of each discrete output frequency dependent on the order. Higher order results are lower in amplitude, with the actual rate of decrease versus order determined by the quality of the mixing circuit. In all cases, the second order responses will have the highest amplitudes:

$$f_1 + f_2$$
$$f_1 - f_2 \quad \text{(actually: } |f_1 - f_2|\text{)}$$

$2f_1$ and $2f_2$ are also second-order outputs, but nearly all practical mixers use a *balanced* design to suppress these outputs, as well as all other even-order harmonics.

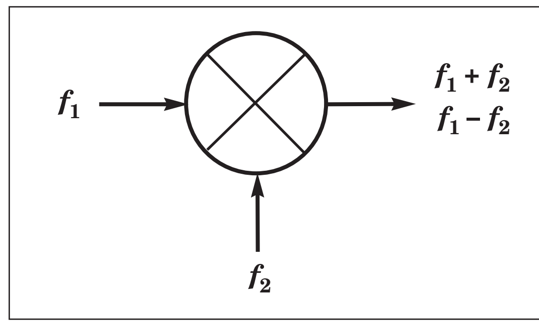

Figure 1 · The frequency translation scheme that is the goal for a frequency mixer.

Figure 1 shows the frequency translation scheme we want to obtain from an ideal mixer. If there are no other outputs, if components are ideal (lossless), then the circuit performs the function of multiplication [1], represented as the trigonometric identity:

$$\cos(\omega_1)\cos(\omega_2) = [\cos(\omega_1 + \omega_2)]/2 + [\cos(\omega_1 - \omega_2)]/2$$

where $\cos(\omega_1)$ and $\cos(\omega_2)$ are the time-domain representations of f_1 and f_2. The 1/2 factors simply show that the input amplitude is divided between the two output terms. In practice, this represents a 6 dB *conversion loss*.

Usually, we want only one of the mixer's outputs, so the unwanted signal must be removed, either by filtering, or by implementing an *image-reject* mixer topology that is actually two mixers with phase shift circuitry that results in a single sum or difference output. Filters have finite stopbands, and image-reject mixers have finite rejection of the unwanted signal. In a sensitive receiver, these imperfect responses may allow strong signals outside the desired passband to be detectable. To minimize this possibility, the relationship of input and output signals must be considered. $f_1 + f_2$ should be chosen so higher-order responses do not fall within the passband of the *intermediate frequency* (IF) filter. Rather that repeat the equations and charts for this type of analysis, References [2, 3] should be consulted.

Real Circuit Performance

An ideal mixer requires perfect switches, as illustrated in Figure 2. In this double-balanced circuit, switches A-D, and B-C are alternately activated at the *local oscillator* frequency, which is the unmodulated signal that determines the amount of frequency difference between input and output signals. In this ideal mixer, the local oscillator signal is not a sine wave, but an ideal square wave with normal and inverted polarity providing the "push-pull" or balanced LO control to the switches.

However, practical circuits do not have zero loss resistance or instantaneous transition times, so an analysis of performance must include these terms. Oxner [4] provides the following description:

An ideal square wave drive will result in switching action according to the Fourier series:

$$F(x) = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\sin[2n-1]\omega t}{[2n-1]}$$

The switching function is derived from this equation as a power function by squaring the first term. Thus the output power deliverable to the output (IF) is:

$$P_{out} = \frac{V_0}{R_L} \qquad \text{or,}$$

$$P_{out} = \frac{V_{in}^2 R_L}{\left[\frac{\pi^2}{4}\left(R_g + R_{SW}\right) + R_L + R_{SW}\right]^2}$$

where R_L is the load impedance, R_g is the internal loss and R_{SW} is device loss (diode junction, or FET R_{DS}).

Conversion efficiency is obtained by the ratio of P_{avg} and P_{out}:

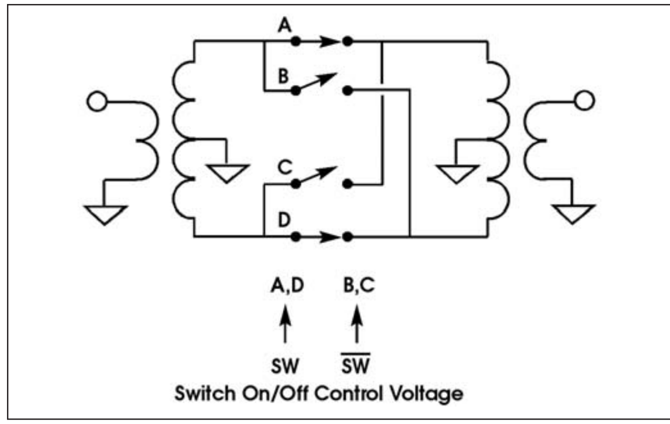

Figure 2 · An ideal mixer has devices (diodes or transistors) that act as perfect switches.

$$L_{conv} = 10\log\frac{\left[\frac{\pi^2}{4}\left(R_g + R_{SW}\right) + R_L + R_{SW}\right]^2}{\pi^2 R_L R_g}$$

Using the above equation, an ideal switching mixer would have a conversion efficiency (in dB) of:

$$L_{conv} = 10\log\frac{4}{\pi^2}$$

which is –3.92 dB. Thus, all mixers will have greater than 3.92 dB conversion loss.

Finally, Oxner provides the following expression that describes the switching function relative to the rise/fall time of the LO switch driver signal (for FET switches):

$$20\log\frac{\left[t_r \omega_{LO}\frac{V_s}{V_c}\right]^2}{8}$$

where V_c is the peak oscillator voltage, V_s is peak signal voltage, and t_r is the rise/fall time of V_c.

References

1. K. McClaning, T. Vito, *Radio Receiver Design*, Noble Publishing, 2000, Ch. 3, "Mixers" (now distributed by SciTech Publishing).

2. L. Besser, R. Gilmore, *Practical RF Circuit Design for Modern Wireless Systems*, Vol. 1, Artech House, 2003, Ch. 3, Section 3.2.6.3 "Spurious responses."

3. R. Carson, *Radio Communications Concepts: Analog*, John Wiley & Sons, 1990, Ch. 9 "Spurious Responses."

4. E. Oxner, "A Commutation Double-Balanced Mixer of High Dynamic Range," *Proceedings, RF Expo East*, 1986, p. 73.